Lesson 21: Solutions to Inequalities with Two Variables

Exercise 1 (5 minutes)

Discuss the two-variable equation in Exercise 1 and the possible solutions represented as ordered pairs. Have students work independently, using their prior knowledge to verify which ordered pairs are solutions to an equation (make a true number sentence).

- **a.** Circle each ordered pair \((x, y)\) that is a solution to the equation \(4x - y \leq 10\).
  - i. \((3, 2)\), \((-1, -14)\), \((0, 0)\), \((1, -6)\)
  - ii. \((5, 10)\), \((0, -10)\), \((3, 4)\), \((6, 0)\), \((4, -1)\)

- **b.** Plot each solution as a point \((x, y)\) in the coordinate plane.

- **c.** How would you describe the location of the solutions in the coordinate plane?
  (Students may struggle to describe the points. Here is one possible description.) The points do not all fall on any one line, but if you drew a line through any two of the points, the others are not too far away from that line.

Ask students to compare their solutions with a partner. Briefly share answers and give students a chance to revise their work or add to their written response to part c. Do not linger on part c - the activity that follows will help to clarify their thinking.

Exercise 2 (10 minutes)

Students should work in groups on part a only. After about 4 minutes, have each group share their solutions and their solution strategies with the entire class. Highlight the different approaches to finding solutions. Most groups will likely start picking a value for either \(x\) or \(y\) and then deciding what the other variable should equal to make the number sentence true.

- **a.** Discover as many additional solutions to the equation \(4x - y \leq 10\) as possible. Organize your solutions by plotting each solution as a point \((x, y)\) in the coordinate plane. Be prepared to share the strategies you used to find your solutions.
  (There are an infinite number of correct answers, as well as an infinite number of incorrect answers. Some sample correct answers are shown.)
  \((1, 1)\), \((1, -3)\), \((-2, 2)\), \((-5, 4)\)

- **b.** Graph the line \(y = 4x - 10\). What do you notice about the solutions to the inequality \(4x - y \leq 10\) and the graph of the line \(y = 4x - 10\)?
  All of the points are either on the line or to the left of / above the line.

- **c.** Solve the inequality for \(y\).
  \(y \geq 4x - 10\)

Scaffolding:
- Pay attention to students that are still struggling to interpret the inequality symbols correctly. Perhaps create a chart or add terms to a word wall that can serve as a reminder to the students.
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d. Complete the sentence, if an ordered pair is a solution to \(4x - y \leq 10\) then it will be located \(\text{on the line or above (or on the left side of)}\) the line \(y = 4x - 10\). Explain how you arrived at your conclusion.

I observed that all the points were on one side of the line, and then I tested some points on the other side of the line and found that all the points I tested from that side of the line were not solutions to the inequality.

Next have the groups complete parts b – d. As they work, circulate around the room answering questions and providing support. Make sure that students reversed the inequality symbol when solving for \(y\) in part c. Discuss the following:

- I noticed some of you wrote that all the points are on the left side of the line and others wrote that all the points are above the line. Are both of those descriptions correct?
- Look now at your answer to part c. When you solved the inequality for \(y\), what does that statement seem to be telling you?
  - It is telling you all the \(y\) values have to be greater than or equal to something related to \(x\).
- Which description would you say best correlates to the inequality we wrote in part c then? Points to the left of the line or points above the line? Why?
  - Points above the line, because when we solved for \(y\), we are describing where the \(y\)-values are in relation to the line, and \(y\)-values are plotted on the vertical axis, so the words above and below are the accurate descriptors.
- How can we depict the entire solution set of ALL the points above the line? When we were working with equations in one variable and graphing our solution set on the number line, how did we show what the solution set was?
  - We colored it darker, or shaded it. So we can just shade in the entire area above the line.
- What about the line itself, is it part of the solution set?
  - Yes.
- What if it wasn’t, what if the inequality was \(y > 4x - 10\)? How could we show that it is all the point except that line?
  - Traditionally what we do is make the line a dashed line instead of a solid line to indicate that the points on the line are not part of the solution set.

Before moving on make sure students understand that any ordered pair in the solution set will be a point \((x, y)\) that is located on or ‘above’ above the line because that is the portion of the coordinate plane where \(y\) is greater than or equal to the difference of \(4x\) and 10.
Example 1 (10 minutes)

The solution to \( x + y = 20 \) is shown on the graph below.

a. Graph the solution to \( x + y \leq 20 \)

b. All points above the line should be shaded.

c. Graph the solution to \( x + y < 20 \)

d. The line should be dashed, and all points above the line should be shaded.
3. Plot the solution sets to the following equations and inequalities on a separate sheet of graph paper.

- a. \(x - y = 10\)
- b. \(x - y < 10\)
- c. \(y > x - 10\)
- d. \(y \geq x\)
- e. \(x \geq y\)
- f. \(y = 5\)
- g. \(y < 5\)
- h. \(x \geq 5\)
- i. \(y \neq 1\)
- j. \(x = 0\)
- k. \(x > 0\)
- l. \(y < 0\)
- m. \(x^2 - y = 0\)
- n. \(x^2 + y^2 > 0\)
- o. \(xy \leq 0\)

Which of the inequalities in this exercise are linear inequalities?
- a–l are linear. m–o are not.

- a-c: Parts b and c are identical. In part a, the solution is the graph of the line.

- d-e: Both solution sets include the line \(y = x\). Part d is the half-plane above the line and part e is the half-plane below the line. When debriefing, ask students to share how they approached part e.

- f-i: These exercises focus on vertical and horizontal boundary lines. Emphasis should be placed on the fact that inequalities like part h are shaded to the left or to the right of the vertical line.

- j-l: These exercises will help students to understand that \(x = 0\) is the y-axis and \(y = 0\) is the x-axis.

- m-o: These exercises can serve as extension questions. For m, a curve separates the plane into two regions. In part n, the solution is the entire coordinate plane except (0,0). And in part o, the solution is all points in quadrants 2 and 4, including both axes and the origin.

The graphical representation of the solution to a two variable linear inequality is called a half plane.
4. Describe in words the half-plane that is the solution to each inequality.

   a. \( y \geq 0 \)
      The half-plane lying above the x-axis and including the x-axis.

   b. \( x < -5 \)
      The half-plane to the left of the vertical line \( x = -5 \), not including the line \( x = -5 \).

   c. \( y \geq 2x - 5 \)
      The line \( y = 2x - 5 \) and the half-plane lying above it.

   d. \( y < 2x - 5 \)
      The half-plane lying below the line \( y = 2x - 5 \).

5. Graph the solution set to \( x < -5 \), reading it as an inequality in one variable and describe the solution set in words. Then graph the solution set to \( x < -5 \) again, this time reading it as an inequality in two variables, and describe the solution set in words.

   Read in one variable: All real numbers less than -5. The graph will have an open circle at the endpoint -5 and extend as a ray to the left of -5 on the number line.

   Read in two variables: all ordered pairs \((x,y)\) such that \( x \) is less than -5. The graph will be a dashed vertical line through \( x = -5 \) and all points to the left of the line will be shaded.

Closing (2 minutes)

- Why is it useful to represent the solution to an inequality with two variables graphically?
- How does graphing the solution set of a one-variable inequality compare to graphing the solution set to a two-variable inequality?

Exit Ticket
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Exit Ticket

What pairs of numbers satisfy the statement: The sum of two numbers is less than 10?

Create an inequality with two variables to represent this situation and graph the solution set.
Exit Ticket Sample Solution

What pairs of numbers satisfy the statement: The sum of two numbers is less than 10? Create an inequality with two variables to represent this situation and graph the solution set.

Let $x =$ one number and let $y =$ another number.

Inequality: $x + y < 10$

Graph the line $y = -x + 10$ using a dashed line and shade below the line.

Problem Set Sample Solutions

1. Match each inequality with its graph. Explain your reasoning.
   a. $2x - y > 6$
      graph 2
   b. $y \leq 2x - 6$
      graph 4
   c. $2x < y + 6$
      graph 3
   d. $2x - 6 \leq y$
      graph 1

Student explanations will vary. Sample response:

I re-arranged each equation and found that they were all the same except for the inequality symbol. The strict inequalities are the dashed lines and the others are solid lines. When solved for $y$, you can decide the shading. Greater than is shaded above the line and less than is shaded below the line.
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2. Graph the solution set in the coordinate plane. Support your answer by selecting two ordered pairs in the solution set and verifying that they make the inequality true.

\[ a. \quad -10x + y > 25 \quad c. \quad y \leq -7.5x + 15 \quad e. \quad 3x < y \]
\[ b. \quad -6 \leq y \quad d. \quad 2x - 8y \leq 24 \quad f. \quad 2x > 0 \]

Solutions are graphed below for parts a, c, and e.

3. Marti sells tacos and burritos from a food truck at the farmers market. She sells burritos for $3.50 each and tacos for $2.00 each. She hopes to earn at least $120 at the farmers market this Saturday.

a. Identify 3 combinations of tacos and burritos that will earn Marti more than $120.

Answers to parts a–c will vary. Answers to part a should be solutions to the inequality \( 3.5x + 2y > 120 \).  

b. Identify 3 combinations of tacos and burritos that will earn Marti exactly $120.

Answers to part b should be solutions to the equation \( 3.5x + 2y = 120 \).  

c. Identify 3 combinations of tacos and burritos that will not earn Marti at least $120.

Answers to part c should not be solutions to the inequality or equation.

\[ d. \quad \text{Graph your answers to parts a–c in the coordinate plane and then shade a half-plane that will contain all possible solutions to this problem.} \]

The graph shown for part d is shown at right. Answers to part a should lie in the shaded half-plane. Answers to part b should lie on the line and answers to part c should lie in the un-shaded half-plane.

e. Create a linear inequality that represents the solution to this problem. Let \( x \) equal the number of burritos that Marti sells and let \( y \) equal the number of tacos that Marti sells.

\[ \text{Part e inequality is } 3.5x + 2y \geq 120. \]

\[ f. \quad \text{Is the points (10, 49.5) a solution to inequality you created in part e? Explain your reasoning.} \]

In part f, the point would not be valid because it would not make sense in this situation to sell a fractional amount of tacos or burritos.