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Lesson 10: Unknown Angle Proofs—Proofs with Constructions

Student Outcome

- Students write unknown angle proofs involving auxiliary lines.

Lesson Notes

On the second day of unknown angle proofs, students incorporate the use of constructions, specifically auxiliary lines, to help them solve problems. In this lesson, students are referring to the same list of facts they have been working with in the last few lessons. The aspect that sets this lesson apart is that necessary information in the diagram may not be apparent without some modification. One of the most common places for an auxiliary line is in diagrams where multiple sets of parallel lines exist. Encourage students to mark up diagrams until the necessary relationships for the proof become more obvious.

Classwork

Opening Exercise (6 minutes)

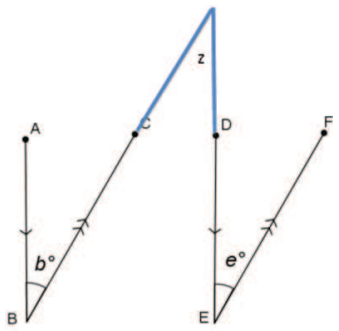
Review the Problem Set from Lesson 9. Then, the whole class works through an example of a proof requiring auxiliary lines.

In the figure to the right, $\overline{AB} \parallel \overline{DE}$ and $\overline{BC} \parallel \overline{EF}$. Prove that $b = e$. (Hint: Extend $\overline{BC}$ and $\overline{ED}$.)

$b = z$ *alt. ∠s*

$z = e$ *alt. ∠s*

$b = e$



Proof:

In the previous lesson, you used deductive reasoning with labeled diagrams to prove specific conjectures. What is different about the proof above?

Adding or extending segments, lines, or rays (referred to as auxiliary) is frequently useful in demonstrating steps in the deductive reasoning process. Once $\overline{BC}$ and $\overline{ED}$ were extended, it was relatively simple to prove the two angles congruent based on our knowledge of alternate interior angles. Sometimes there are several possible extensions or additional lines that would work equally well.

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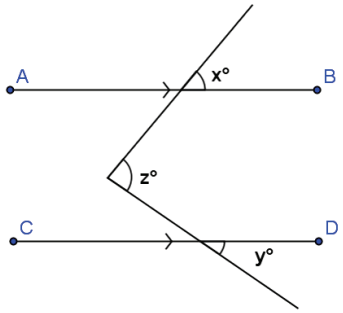
For example, in this diagram, there are at least two possibilities for auxiliary lines. Can you spot them both?



Given: $\overline{AB} \parallel \overline{CD}$.

Prove: $z = x + y$.

Discussion (7 minutes)

Students explore different ways to add auxiliary lines (construction) to the same diagram.

Here is one possibility:

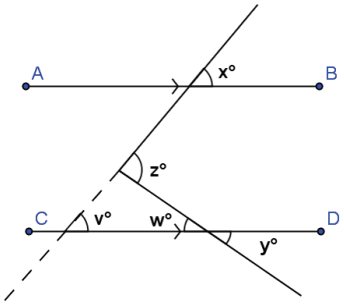
Given: $\overline{AB} \parallel \overline{CD}$.

Prove: $z = x + y$.

Extend the transversal as shown by the dotted line in the diagram. Label angles v and w, as shown.

What do you know about v and x?

About angles w and y? How does this help you?



Write a proof using the auxiliary segment drawn in the diagram to the right.

$z = v + w$	*ext. ∠s*
$x = v$	*corr. ∠s*
$y = w$	*vert. ∠s*

$z = x + y$

MP.7

Another possibility appears here:

Given: $\overline{AB} \parallel \overline{CD}$.
Prove: $z = x + y$.

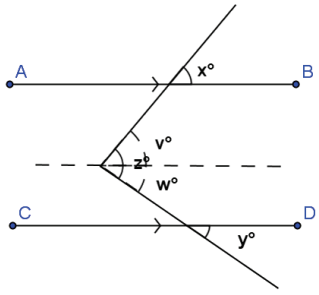
Draw a segment parallel to $\overline{AB}$ through the vertex of angle z. This divides $\angle z$ into angles v and w.

What do you know about angles v and x?

They are equal in measure since they are corresponding angles of parallel lines.

About angles w and y? How does this help you?

They are also equal in measure since they are corresponding angles of parallel lines.



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Write a proof using the auxiliary segment drawn in this diagram. Notice how this proof differs from the one above.

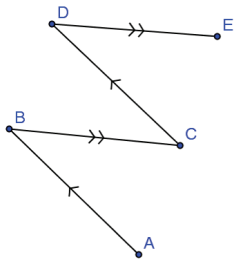
$x = v$ *corr. ∠s*

$y = w$ *corr. ∠s*

$z = v + w$

Examples (25 minutes)

1. **In the figure, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DE}$.**
Prove that $\angle ABC = \angle CDE$.
(Where will you draw an auxiliary segment?)

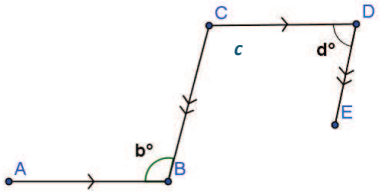


$\angle ABC = \angle BCD$ *alt. ∠s*

$\angle BCD = \angle CDE$ *alt. ∠s*

$\angle ABC = \angle CDE$

2. **In the figure, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DE}$.**
Prove that $b + d = 180°$.



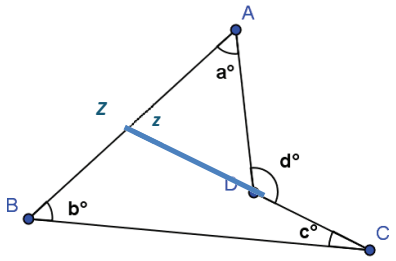
Label c

$b = c$ *alt. ∠s*

$c + d = 180°$ *int. ∠s*

$b + d = 180°$

3. **In the figure, prove that $d = a + b + c$.**



Label Z and z.

$z = b + c$ *ext. ∠s*

$d = z + a$ *ext. ∠s*

$d = a + b + c$

Exit Ticket (5 minutes)

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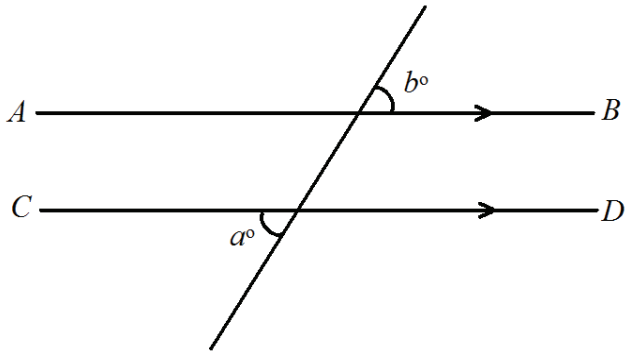
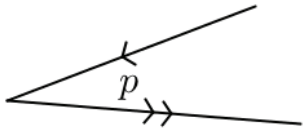
Exit Ticket

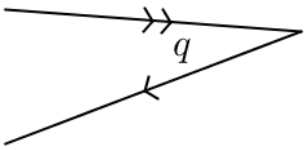
Write a proof for each question.

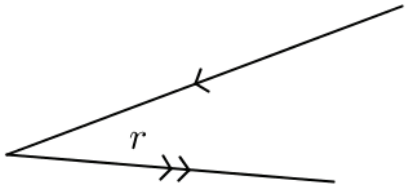
1. In the figure, $\overline{AB} \parallel \overline{CD}$. Prove that $a = b$.



2. Prove $p \cong r$







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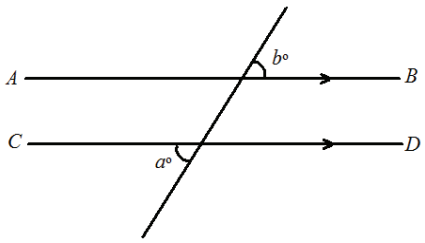
Exit Ticket Sample Solutions

Write a proof for each question.

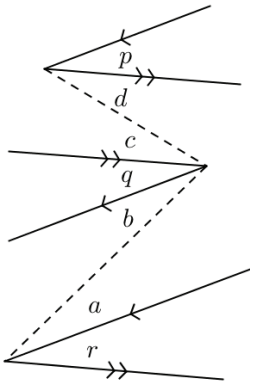
1. **In the figure, $\overline{AB} \parallel \overline{CD}$. Prove that $a = b$.**



Write in angles c and d.

$a = c$	*vert. ∠s*
$c = d$	*alt. ∠s*
$d = b$	*vert. ∠s*
$a = b$	

2. **Prove $p \cong r$.**



Mark angles a, b, c, and d.

$p + d \cong c + q$	*Alt. Int. ∠s*
$d \cong c$	*Alt. Int. ∠s*
$p \cong q$	*substitution*
$q + b \cong a + r$	*Alt. Int. ∠s*
$a \cong b$	*Alt. Int. ∠s*
$q \cong r$	*substitution*
$p \cong r$	

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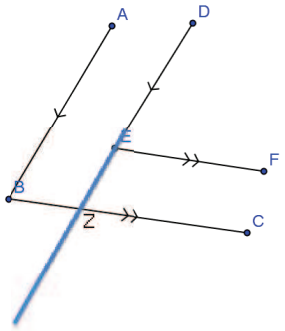
Problem Set Sample Solutions

1. **In the figure, $\overline{AB} \parallel \overline{DE}$ and $\overline{BC} \parallel \overline{EF}$.**
   Prove that ∠ABC = ∠DEF.



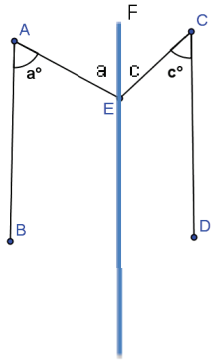
Extend DE through BC, mark the intersection with BC as Z

$\angle ABC = \angle EZC$ *corr. ∠s*

$\angle EZC = \angle DEF$ *corr. ∠s*

$\angle ABC = \angle DEF$

2. **In the figure, $\overline{AB} \parallel \overline{CD}$.**
   Prove that ∠AEC = a + c.



Draw in line through E parallel to AB and CD,

add point F.

$\angle BAE = \angle AEF$ *alt. ∠s*

$\angle DCE = \angle FEC$ *alt. ∠s*

$\angle AEC = a + c$