Lesson 16: Solving and Graphing Inequalities Joined by “And” or “Or”

Student Outcomes

- Students solve two inequalities joined by “and” or “or,” then graph the solution set on the number line.

Classwork

Exercise 1 (5 minutes)

Do questions 1–3 and as much of 4–5 as time permits depending on the level of your students. You can also present the challenge problem given below if time allows.

a. Solve \( w^2 = 121 \), for \( w \). Graph the solution on a number line.

b. Solve \( w^2 < 121 \), for \( w \). Graph the solution on a number line and write the solution set as a compound inequality.

c. Solve \( w^2 \geq 121 \) for \( w \). Graph the solution on a number line and write the solution set as a compound inequality.

d. Quickly solve \( (x + 7)^2 = 121 \), for \( x \). Graph the solution on a number line.

e. Use your work on (d) to quickly graph the solution on a number line to each inequality below.

i. \( (x + 7)^2 < 121 \)

ii. \( (x + 7)^2 \geq 121 \)
Extension

Use the following to challenge students who finish early.

a. Poindexter says that \((a + b)^2\) equals \(a^2 + 2ab + b^2\). Is he correct?

b. Solve \(x^2 + 14x + 49 < 121\), for \(x\). Present the solution graphically on a number line.

Exercises 2–3 (7 minutes)

Give students 5 minutes to work on Exercises 2 and 3. Then, discuss the results as a class. Students are applying their knowledge from the previous lesson to solve an unfamiliar type of problem.

Exercise 2

Consider the compound inequality \(-5 < x < 4\)

a. Rewrite the inequality as a compound statement of inequality.
   \[x > -5 \text{ and } x < 4\]

b. Write a sentence describing the possible values of \(x\).
   \(x\) can be any number between \(-5\) and 4.

c. Graph the solution set on the number line below.

Exercise 3

Consider the compound inequality \(-5 < 2x + 1 < 4\)

a. Rewrite the inequality as a compound statement of inequality.
   \[2x + 1 > -5 \text{ and } 2x + 1 < 4\]

b. Solve each inequality for \(x\). Then, write the solution to the compound inequality.
   \[x > -3 \text{ and } x < \frac{3}{2} \text{ OR } -3 < x < \frac{3}{2}\]

c. Write a sentence describing the possible values of \(x\).
   \(x\) can be any number between \(-3\) and \(\frac{3}{2}\)

d. Graph the solution set on the number line below.
Review Exercise 3 with students to demonstrate how to solve it without re-writing it:

- A friend of mine suggested I could solve the inequality as follows. Is she right?

\[
-5 < 2x + 1 < 4 \\
-5 - 1 < 2x + 1 - 1 < 4 - 1 \\
-6 < 2x < 3 \\
-3 < x < \frac{3}{2}
\]

Encourage students to articulate their thoughts and scrutinize each other’s reasoning.

Point out to students that solving the two inequalities did not require any new skills. They are solved just as they learned in previous lessons.

Have students verify their solution by filling in a few test values.
Remind students that the solution can be written two ways: \( x > -3 \) and \( x < 3/2 \)

**OR** \(-3 < x < \frac{3}{2}\)

**Exercises 4–5 (6 minutes)**

Give students 5 minutes to work on Exercises 4 and 5. Then, review the results as a class. Again point out to students that solving the two inequalities did not require any new skills. They are solved just as they learned in previous lessons. Have students verify their solution by filling in a few test values.

**Exercise 4**

Given \( x < -3 \) or \( x > -1 \)

a. What must be true in order for the compound inequality to be a true statement?

   *One of the statements must be true, so either \( x \) has to be less than \(-3\), or it has to be greater than \(-1\), (in this case it is not possible that both are true.)*

b. Write a sentence describing the possible values of \( x \).

   \( x \) can be any number that is less than \(-3\) or any number that is greater than \(-1\)

c. Graph the solution set on the number line below.

   ![Number Line](image)

**Scaffolding for Diverse Learners:**

- Remind students that when an inequality is multiplied or divided by a negative number, the direction of the inequality changes.
Lesson 16

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Exercise 5

Given: \( x + 4 < 6 \) or \( x - 1 > 3 \)

a. Solve each inequality for \( x \). Then, write the solution to the compound inequality.
   \[ x < 2 \text{ or } x > 4 \]

b. Write a sentence describing the possible values of \( x \).
   \( x \) can be any number that is less than 2 or any number that is greater than 4.

c. Graph the solution set on the number line below.

![Number line with solutions marked]

Exercise 6 (15 minutes)

Have students work the exercises individually or with partners or small groups. Circulate around the room monitoring progress and offering guidance as needed. Make sure students are attending to the detail of correctly using open and closed endpoints.

Solve each compound inequality for \( x \), and graph the solution on a number line.

a. \( x + 6 < 8 \) and \( x - 1 > -1 \)
   \[ x < 2 \text{ and } x > 0 \Rightarrow 0 < x < 2 \]

b. \( -1 \leq 3 - 2x \leq 10 \)
   \[ x \geq -\frac{7}{2} \text{ and } x \leq 2 \Rightarrow -\frac{7}{2} \leq x \leq 2 \]

c. \( 5x + 1 < 0 \) or \( 8 \leq x - 5 \)
   \[ x < -\frac{1}{5} \text{ or } x \geq 13 \]

d. \( 10 > 3x - 2 \) or \( x = 4 \)
   \[ x < 4 \text{ or } x = 4 \Rightarrow x \leq 4 \]

![Number line with solutions marked for each inequality]
Debrief the exercise with the following questions:

- Look at the solution to question \( f \) closely. Remind students that both statements must be true. Therefore, the solution is only \( x = 6 \).
- How would the solution to question \( e \) change if the “and” was an “or”? Let this discussion lead in to Exercise 7.

**Exercise 7 (10 minutes)**

Have students work in groups to answer the questions. Students are exploring variations of previously seen problems. After completing the exercises ask students to articulate how the problems differed from most of the other examples seen thus far.

Solve each compound inequality for \( x \) and graph the solution on a number line. Pay careful attention to the inequality symbols and the ‘and’ or ‘or’ statements as you work.

- \( a. \quad 1 + x > -4 \) or \( 3x - 6 > -12 \)
  \[ x > -5 \]
- \( b. \quad 1 + x > -4 \) or \( 3x - 6 < -12 \)
  \[ x \text{ can be any real number} \]
- \( c. \quad 1 + x > 4 \) and \( 3x - 6 < -12 \)
  \[ x > 3 \text{ or } x < -2 \]
Have early finishers explore further with the following:

- Is it possible to write a problem separated by “or” that is no solution?
- Is it possible to have a problem separated by “and” that is all real numbers?

**Closing (2 minutes)**

For the first problem, students may have written the solution as $x > -5$ or $x > -2$. Look at the graph as a class, and remind them that the solution is the all of the numbers included in either of the two solution sets (or the union of the two sets). Lead them to the idea that the solution is $x > -5$.

For the second problem, the two graphs overlap and span the entire number line. Lead them to the idea that the solution is all real numbers. Have students fill in a few test values to verify that any number will work.

For the third problem, the two graphs do not overlap. Remind them that the solution set is only the values that are in both of the individual solution sets. There is no number that will make both statements true. Lead them to the idea that there is no solution.

Read the questions at the end of the exploration and give students a few minutes to summarize their thoughts on the work in Exercise 7 independently. Call for a few volunteers to read their solutions.

**Exit Ticket**
Lesson 16: Solving and Graphing Inequalities Joined by “And” or “Or”

Exit Ticket

1. Solve each compound inequality for $x$ and graph the solution on a number line.
   a. $9 + 2x < 17$ or $7 - 4x < -9$
   b. $6 \leq \frac{x}{2} \leq 11$

2. a. Give an example of a compound inequality separated by “or” that has a solution of all real numbers.
   b. Take the example from (a) and change the “or” to an “and.” Explain why the solution set is no longer all real numbers. Use a graph on a number line as part of your explanation.
Exit Ticket Sample Solutions

1. Solve each compound inequality for $x$ and graph the solution on a number line.
   a. $9 + 2x < 17$ or $7 - 4x < -9$
      $x < 4$ or $x > 4$ OR $x \neq 4$
   b. $6 \leq \frac{x}{2} \leq 11$
      $12 \leq x \leq 22$

2. a. Give an example of a compound inequality separated by "or" that has a solution of all real number.
   Sample response: $x > 0$ or $x < 2$
   b. Take the example from (a) and change the “or” to an “and.” Explain why the solution set is no longer all real numbers. Use a graph on a number line as part of your explanation.
   Sample response: $x > 0$ and $x < 2$
   In the first example, only one inequalities needs to be true to make the compound statement true. Any number selected is either greater than 0 or less than 2 or both. In the second example, both inequalities must be true to make the compound statement true. This restricts the solution set to only numbers between 0 and 2.

Problem Set Sample Solutions

Solve each inequality for $x$ and graph the solution on a number line.

1. $x - 2 < 6$ or $\frac{x}{3} > 4$
   $x < 8$ or $x > 12$

2. $-6 \leq \frac{x + 1}{4} < 3$
   $-25 \leq x < 11$

3. $5x \leq 21 + 2x$ or $3(x + 1) \geq 24$
   $x \leq 7$ or $x \geq 7$ all real numbers

4. $5x + 2 \geq 27$ and $3x - 1 < 29$
   $x \geq 5$ and $x < 10$ $5 \leq x < 10$

5. $0 \leq 4x - 3 \leq 11$
   $\frac{3}{4} \leq x \leq 7$

6. $2x > 8$ or $-2x < 4$
   $x > 4$ or $x > -2$ $x > -2$

7. $8 \geq 2(x - 9)$ $\geq -8$
   $5 \leq x \leq 13$

8. $4x + 8 > 2x - 10$ or $\frac{1}{2}x - 3 < 2$
   $x > -9$ or $x < 15$ all real numbers
9. \( 7 - 3x < 16 \quad \text{and} \quad x + 12 < -8 \)  
\( x > -3 \quad \text{and} \quad x < -20 \rightarrow \text{no solution} \)

10. If inequalities question 8 were joined by “and” instead of “or,” what would the solution set become?  
\( -9 < x < 15 \)

11. If the inequalities in question 9 were joined by “or” instead of “and,” what would the solution set become?  
\( x > -3 \quad \text{or} \quad x < -20 \)